For $n=72$, $\cos DOE=.99559$, $DOE=5° 29'$, an error of $29'$.

For large values of n the error is much too great for any purpose.

Also solved by *J. E. SANDERS*, Hackney, Ohio.

CALCULUS.

165. Proposed by CAPT. T. C. DICKSON, Ordnance Department, United States Army, Washington, D. C.

Solve by integration the differential equation

$$\frac{d^2\xi}{dt^2}+\frac{A}{B}\left(\frac{d\xi}{dt}\right)^2-\frac{C}{B}=0,$$

in which A, B, C are given functions of ξ, but independent of t.

Solution by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

In view of the nature of the coefficients, we regard ξ as the independent variable and t the dependent, the formulae of transformation being

$$\frac{d\xi}{dt}=1\div\frac{dt}{d\xi},\quad \frac{d^2\xi}{dt^2}=-\frac{d^2t}{d\xi^2}\div\left(\frac{dt}{d\xi}\right)^3.$$

The given equation thus becomes

$$\frac{d^2t}{d\xi^2}-\frac{A}{B}\frac{dt}{d\xi}+\frac{C}{B}\left(\frac{dt}{d\xi}\right)^3=0.$$

Set $dt/d\xi=y$, whence $t=\int yd\xi$. Then $\frac{dy}{d\xi}-\frac{A}{B}y+\frac{C}{B}y^3=0.$

Divide by y^3 and set $z=y^{-2}$. The resulting differential equation

$$\frac{dz}{d\xi}+\frac{2A}{B}z-\frac{2C}{B}=0$$

is linear. By the usual method, we get

$$z=2e^{-\lambda}\left(\int\frac{C}{B}e^{\lambda}\,d\xi+k\right),\quad \lambda\equiv2\int\frac{A}{B}d\xi.\quad \therefore\ t=\int z^{-\frac{1}{2}}d\xi.$$

169. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Find the value of y from the Eulerian equation

$$y=\int\frac{dx}{(x+\sqrt{3})\sqrt[3]{(x^2+1)}}.$$

Solution by G. B. M. ZERR. A. M., Ph. D., Parsons, W. Va.

Let $x=\dfrac{\sqrt{3}(z+1)}{z-1}$, $z^3-1=u^3$.

$$\therefore \int \frac{dx}{(x+\sqrt{3})\sqrt[3]{(x^2+1)}} = -\int \frac{dz}{z\sqrt[3]{[4(z^3-1)}} = -\int \frac{udu}{\sqrt[3]{4(u^3+1)}}.$$

$$\therefore\ y=\frac{1}{3\sqrt[3]{4}}\int\frac{du}{1+u}-\frac{1}{6\sqrt[3]{4}}\int\frac{(2u-1)du}{1-u+u^2}-\frac{1}{2\sqrt[3]{4}}\int\frac{du}{1-u+u^2}.$$

$$\therefore\ y=\frac{1}{6\sqrt[3]{4}}\log\left(\frac{(1+u)^2}{1-u+u^2}\right)-\frac{1}{\sqrt{3}\sqrt[3]{4}}\tan^{-1}\left(\frac{2u-1}{\sqrt{3}}\right),$$

where $u=\dfrac{\sqrt[3]{[6\sqrt{3}(x^2+1)]}}{x-\sqrt{3}}$.

170. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

Find the center-locus of conics having 4-point contact with a given conic at a given point. Show that the conic of minimum eccentricity is given by $e^4\tan^2\varphi+4e^2-4=0$, where e is its eccentricity, and φ is the angle which the linear center-locus above makes with the normal to the curve at the point.

Solution by WILLIAM HOOVER, Ph. D., Professor of Mathematics in the State University, Athens, Ohio.

The coördinates of the center of any conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0.....(1), \text{ are}$$

$$x_1=\frac{hf-bg}{ab-h^2}.....(2),\ y_1=\frac{gh-af}{ab-h^2}.....(3),$$

and the eccentricity is given by

$$e^4+\frac{(a-b)^2+4h^2}{ab-h^2}(e^2-1)=0.....(4).$$

If the tangent and normal to the given curve be the axes of abscissas and ordinates, the equation of the conic having 4-point contact with the given conic is of the form

$$ax^2+2hxy+by^2+2gx-\lambda x^2=0.....(5), \text{ or, } (a-\lambda)x^2+2hxy+by^2+2gx=0.....(6).$$

Comparing (1) and (6), $a=a-\lambda$, $f=0$, $c=0$.....(7), and (2) and (3) become

$$x_1=-\frac{bg}{(a-\lambda)b-h^2}.....(8),\quad y_1=\frac{gh}{(a-\lambda)b-h^2}.....(9).$$

(9)÷(8) gives $y_1-(h/b)x_1$.....(10), the locus-center. Thus the "slope" is